Designing PID (+ notches) controllers using loopshaping

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Preparation

Some of the exercises are accompanied by MATLAB-scripts. Follow the next steps to use them. Always explain which steps have lead you to your solutions!

Extract all the files contained in the supplementary `.zip` file to a new folder on your computer. Start MATLAB, change your working directory to the folder (do not use 'Add to Path'!) that you've created and closely follow the instructions. Mostly, these scripts need some minor modifications in order to match the problem solved with your own calculations.

Introduction

- Your ability to complete this assignment is important prior knowledge for the courses 'Robust Control' and 'Control engineering' in Q2.
- You are expected to answer the questions listed below in a (PDF) document. You may use some form of LaTeX, an office text writer or write your answers by hand,[1] as long as your answers are presented in a presentable manner.

[1]Though we will ask you to include figures obtained with MATLAB.

1 Simple mass

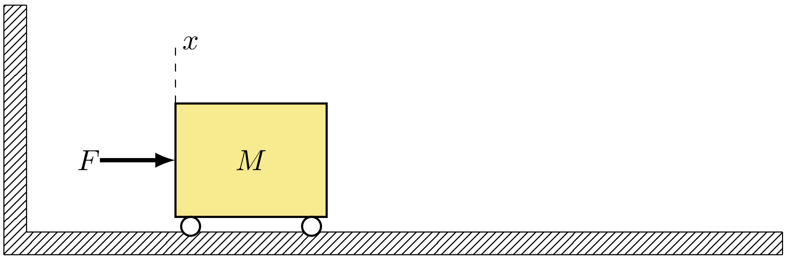

Figure 1: Single Mass

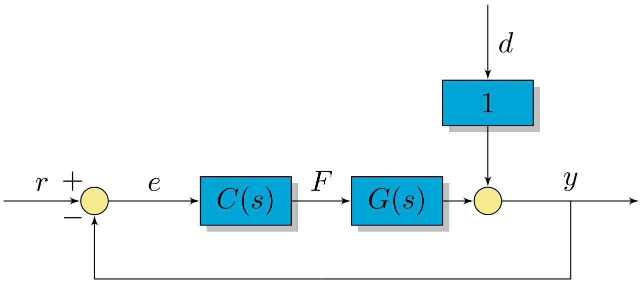

Figure 2: Simple feedback interconnection

Consider the system depicted in Figure 1 with $M = 0.1[kg]$ and the feedback configuration in Figure 2. In this assignment we would like to see that the cart follows a certain reference position $r(t) = x_s(t)$, $G(s) = \frac{1}{Ms^2}$ and there is no measurement disturbance $d(t) = 0$.

1. Compute the pole locations if we use proportional feedback $C(s) = k_p = 100$. Compute a step response, Bode of the loop transfer function, and Nyquist of the loop transfer. What would happen if you vary the gain? What can you conclude?
2. Find the gains of a PD controller of the form $C(s) = k_p + k_d s$. Choose k_p and k_d such that the system has a Gain Cross-over frequency (ω_{gc}) of 5 Hz and a Phase margin of 45 degrees.
3. Evaluate the step-response of the closed loop system. Make a Nyquist diagram and Bode plot of the loop transfer function. What can you conclude?
4. Do the same for a ω_{gc} of 50 Hz and explain your findings.
5. Add a time-delay of 0.01s and evaluate the performance of the two systems (systems with different ω_{gc}) and explain your findings
6. In the case of ω_{gc} equals 5Hz. What is the bound on the amount of time-delay that you can add before the system becomes unstable?

2 4^{th} order system, Position x_1

Consider the setup of Figure 3 and the feedback configuration in Figure 2. We assume the following coefficients:

$$M_1 = M_2 = 1, K = 200, C = 0.1$$

In this exercise we will study $G(s) = \frac{X_1(s)}{F(s)} = \dfrac{M_2 s^2 + Cs + K}{(M_1 s^2 + Cs + K)(M_2 s^2 + Cs + K) - (Cs + K)^2}$ and $d = 0$.

1. Design a PD feedback controller $C(s) = k_p \dfrac{\frac{k_d}{k_p}s + 1}{\frac{k_d}{k_p 100}s + 1}$ to control the position $y = x_1$, assuming that x_1 can be used for feedback. Design the controller such that the system has a reasonable bandwidth of

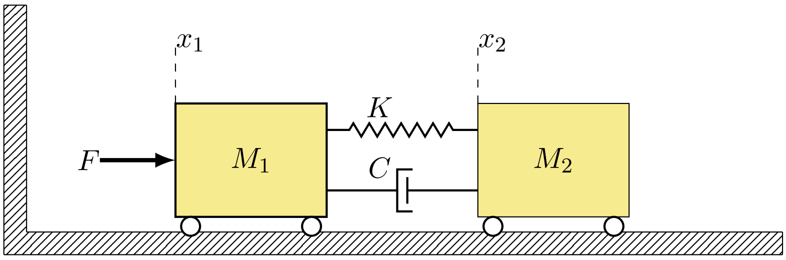

Figure 3: Double Mass-Spring

approximately 1Hz, assuring that the system remains stable and well-damped. Motivate your choice. (use Nyquist and Bode plots of the loop transfer function to design your controller, show the different margins)

2. Evaluate the influences of the two controller parameters on the step response of the closed-loop system. Also indicate the effect on each of the following performance specifications:
   Bandwidth, stability margin, steady-state error.

3. For the designed controller, relate the performance specifications from the previous question to the sensitivity and complementary sensitivity functions?

3 4^{th} order system, Position x_2

Consider again the setup of Figure 3 and the feedback configuration in Figure 2. Assume the following specifications:

$$M_1 = M_2 = 1, K = 200, C = 0.1$$

In this exercise we will study $G(s) = \frac{X_2(s)}{F(s)} = \frac{K}{(M_1 s^2 + Cs + K)(M_2 s^2 + Cs + K) - (Cs + K)^2}$ and $d = 0$ (for the first couple of questions).

1. Design a PD feedback controller with $C(s) = k_p \frac{\frac{k_d}{k_p} s+1}{\frac{k_d}{k_p * 100} s+1}$ to control the position $y = x_2$, assuming that x_2 can be used for feedback. Design the controller such that the system has the maximum possible bandwidth, assuring that the system remains stable and well-damped. Motivate your choices (use Bode and Nyquist of the loop transfer function to design your controller)

2. Design a controller such that the system has a bandwidth of approximately 1Hz, assuring that the system remains stable and well-damped. Motivate your choices. (use Bode and Nyquist of the loop transfer function to design your controller) (Hint: use a notch)

3. We have a persistent measurement disturbance of the following form: $d = sin(t)$. Augment the designed controller with an inverted notch which mitigates this disturbance with a factor 100 with respect to the controller design in the previous question. Motivate you choices (use a sensitivity plot, Nyquist, a time domain plot in the situation with and without this additional controller). Based on a comparison of the two sensitivity plots, explain the waterbed effect.

4. Assume we have access to the disturbance signal d. Design a feedforward filter that directly eliminates the effect of the disturbance $d(t)$. (HINT, can be a constant gain)

References